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**PATENT APPLICATION**

**ATTORNEY DOCKET NO.** 10004812-1

AF/3627 /B

**IN THE**
**UNITED STATES PATENT AND TRADEMARK OFFICE**

#13 Appeal B.
11-7-03
S.E.

**Inventor(s):** Steve Kakouros et al.

**Confirmation No.:** 9467

**Application No.:** 09/608,057

**Examiner:** Cuff, Michael A.

**Filing Date:** June 30, 2000

**Group Art Unit:** 3627

**Title:** SPOT MARKET-BASED INVENTORY PLANNING

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**Mail Stop Appeal Brief-Patents**
**Commissioner For Patents**
**PO Box 1450**
**Alexandria, VA 22313-1450**

**TRANSMITTAL OF APPEAL BRIEF**

**Sir:**

Transmitted herewith in **triplicate** is the Appeal Brief in this application with respect to the Notice of Appeal filed on **Sep. 17, 2003**.

The fee for filing this Appeal Brief is (37 CFR 1.17(c)) $320.00.

**(complete (a) or (b) as applicable)**

The proceedings herein are for a patent application and the provisions of 37 CFR 1.136(a) apply.

( ) (a) Applicant petitions for an extension of time under 37 CFR 1.136 (fees: 37 CFR 1.17(a)-(d) for the total number of months checked below:

( ) one month $110.00
( ) two months $410.00
( ) three months $930.00
( ) four months $1450.00

( ) The extension fee has already been filled in this application.

**(X)** (b) Applicant believes that no extension of term is required. However, this conditional petition is being made to provide for the possibility that applicant has inadvertently overlooked the need for a petition and fee for extension of time.

Please charge to Deposit Account **08-2025** the sum of **$320.00**. At any time during the pendency of this application, please charge any fees required or credit any over payment to Deposit Account 08-2025 pursuant to 37 CFR 1.25. Additionally please charge any fees to Deposit Account 08-2025 under 37 CFR 1.16 through 1.21 inclusive, and any other sections in Title 37 of the Code of Federal Regulations that may regulate fees. A duplicate copy of this sheet is enclosed.

**(X)** I hereby certify that this correspondence is being deposited with the United States Postal Service as first class mail in an envelope addressed to: Commissioner for Patents, Alexandria, VA 22313-1450. Date of Deposit: **Oct. 27, 2003**

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( ) I hereby certify that this paper is being transmitted to the Patent and Trademark Office facsimile number________ on ________

Number of pages:

Typed Name: **Edouard Garcia**

Signature:

Respectfully submitted,

**Steve Kakouros et al.**

By

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IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

Applicant : Steve Kakouros et al.
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Commissioner for Patents
PO Box 1450
Alexandria, VA 22313-1450



# APPEAL BRIEF

## I. Real Party in Interest

The real party in interest is Hewlett-Packard Development Company, L.P., a Texas Limited Partnership having its principal place of business in Houston, Texas.

## II. Related Appeals and Interferences

Appellant is not aware of any related appeals or interferences that will directly affect or be directly affected by or have a bearing on the Board's decision in the pending appeal.

## III. Status of Claims



Claims 1- 21 are pending.

Claims 1-21 stand rejected under 35 U.S.C. § 103(a) over Garg (U.S. 6,144,945) in view of Salvo (U.S. 6,314,271).

Appellant appeals all rejections of the pending claims 1-21.

**CERTIFICATE OF MAILING**

I hereby certify that this correspondence is being deposited with the United States Postal Service as First Class Mail in an envelope addressed to: Commissioner for Patents, PO Box 1450, Alexandria, VA 22313-1450 on:

October 27, 2003
Date

(Signature of person mailing papers)

Edouard Garcia
(Typed or printed name of person mailing papers)

## IV. Status of Amendments

The amendments filed March 20, 2003, have been entered and acted upon by the Examiner.

No amendments were filed after the final rejection dated Jun 17, 2003.

## V. Summary of Invention

The invention defined in the claims on appeal is exemplified by embodiments that plan a safety stock level to cover uncertainty in demand over an exposure period with a desired service level based at least in part upon product availability from a spot market. The invention provides additional flexibility relative to prior inventory planning approaches by accommodating supply availability from spot markets, which are characterized by relatively uncertain price information but relatively certain delivery information. By incorporating spot market supply information into an inventory planning model, the invention enables asset managers to take advantage of spot market supplies to reduce inventory levels and lower overall product supply costs relative to business models that rely solely on non-spot market supplies. In particular, the invention provides an inventory planning scheme that enables an asset manager to cover uncertainty in future end customer demand with a safety stock level that is less than the safety stock level required to cover expected demand with a desired service level when supply is available only from non-spot market sources. In this way, an asset manager may trade some certainty in product price for a greater certainty in product delivery times to reduce overall product costs by reducing the level of safety stock kept on hand. In some circumstances, per unit prices of products supplied by spot market sources may be higher than the prices of comparable products and components supplied by non-spot market sources. However, depending upon market conditions, overall supply chain costs may be lowered by reducing safety stock levels (and, consequently, lowering total safety stock costs), and supplying from spot market sources the fraction of unmet actual demand needed to reach target service levels.

Claims 1-21 cover methods of planning a safety stock level to cover uncertainty in demand over an exposure period with a desired service level based at least in part upon

product availability from a spot market. Embodiments within the scope of claims 1-21 are described with reference to FIGS. 5 and 6 at page 8 line 7, through page 10, line 23.

Claims 2-17 specifically are drawn to methods in which a maximum safety stock level of the product to cover uncertainty in demand over the exposure period with the desired service level is estimated based upon product availability from a non-spot market supply, and an optimal safety stock level is estimated by reducing the maximum safety stock level based upon product availability from a spot market supply. Embodiments within the scope of these claims are described with reference to FIGS. 5 and 6 at page 8 line 7, through page 10, line 23. An example of the estimated total cost recited in claims 8-13 is defined on page 8, line 23, and shown in FIG. 5 as the curve labeled $C_{TOTAL}(q_{SPOT})$. An embodiment incorporating the stochastic simulation recited in claim 14 is described with reference to FIG. 7 at page 10, line 24, through page 11, line 9. An embodiment incorporating the step of ordering the optimal safety stock level recited in claim 15 is shown in FIG. 9, where block 88 corresponds to the optimal safety stock level ordering step (see page 13, lines 18-20). An example of the product level recited in claims 16 and 17 that is needed to meet demand above the optimal safety stock level is shown by the bracket labeled $q_{SAFEETY, OPTIMAL}$ in FIG. 5. An embodiment incorporating the web site recited in claims 17-19 is described with reference to FIG. 10 at page 13, line 29, through page 15, line 7.

## VI. Issues

Sole Issue: Whether claims 1-21 are patentable under 35 U.S.C. § 103(a) over Garg (U.S. 6,144,945) in view of Salvo (U.S. 6,314,271)?

## VII. Grouping of Claims

For each ground of rejection that Appellant contests herein and applies to more than one claim, such additional claims, to the extent separately identified and argued below, do not stand or fall together.

## VIII. Argument

**Sole Issue: Whether claims 1-21 are patentable under 35 U.S.C. § 103(a) over Garg (U.S. 6,144,945) in view of Salvo (U.S. 6,314,271)?**

The Examiner has rejected claims 1-21 under 35 U.S.C. § 103(a) over Garg (U.S. 6,144,945) in view of Salvo (U.S. 6,314,271).

### A. Independent claim 1

Independent claim 1 recites:

> 1. An inventory planning method, comprising planning a safety stock level to cover uncertainty in demand over an exposure period with a desired service level based at least in part upon product availability from a spot market.

### 1. The Disclosure of Garg

In the final Office action dated June 17, 2003, the Examiner asserted that:

> Garg et al. shows all of the limitations of the claims except for specifying that the planning of a safety stock level based on the use of product availability from the spot market and non-spot market.
>
> Garg et al. shows, figures 1 and 2, an inventory system with an order generation module 34 which can transmit orders via the Internet (includes web sites). Figure 2 and top of column 8 show a process loop with the following inputs: $\mu$ = mean demand/period (estimation of demand), $\delta$ = standard deviation

> of demand/period (uncertainty in demand/stochastic simulation), l = lead time, r = review period length (exposure period), h = holding cost/unit/period (price inherent), p = shortage penalty/unit/period, K = ordering unit, f = fill rate, A = Average cost/period (total cost inherent), EI = expected on-hand inventory, EB = expected back order level, and SS = safety stock level. When an optimal solution is achieved, then the solution including average cost per period, expected on-hand inventory, expected back order level, and safety stock level is output at 208.

In the Response of March 20, 2003, Applicants explained that "Garg's process for optimizing the parameters for the (s, S) inventory policy does not have product pricing information as an input." In response to these arguments, the Examiner indicated that (emphasis added):

> Applicant asserts that it would not have been obvious to combine the Garg and Salvo inventions "because Garg's process for optimizing the parameters for the inventory policy does not have product pricing information as an input." The examiner does not concur. <u>Product pricing is included in the parameters h (holding cost/unit/period) and A (average cost/period).</u>

As explained in detail below, the Examiner's understanding of Garg's teaching, as expressed in the above-quoted paragraph from the final Office action dated June 17, 2003, was patently flawed.

<u>a. Product Pricing Is *Not* Included In The Holding Cost Parameter (h)</u>

In FIG. 2, Garg shows the complete set of parameter values 201 that are input into his inventory optimization process. Among these inputs 201, the Examiner has asserted that product pricing is included in the input parameter h, which Garg defines as "the holding cost per period" (col. 7, line 47). Holding cost for a product is a well-defined parameter in the field of operations research. In general, holding costs are the costs "associated with keeping stock over time" (see, e.g., the middle of page 2 of the reference "OR-Notes" by J. E. Besley; attached following page 6 of this Response). Accordingly, holding costs are not linked to the current market price of the product. Indeed, this fact is verified by reviewing the types of costs that are included in determining holding cost (see, e.g., the middle of page 2 of the reference "OR-Notes" by J. E. Besley; attached):

- storage costs
- rent/depreciation

- labor
- overheads (e.g., heating, lighting, security)
- money tied up (loss of interest, opportunity cost)
- obsolescence costs (if left with stock at the end of product life)
- stock deterioration (lose money if product deteriorates whilst held)
- theft/insurance

Therefore, contrary to the Examiner's assertion quoted above, product pricing is *not* included in the holding cost parameter (h).

b. Product Pricing Is *Not* Included In The Average Cost Parameter (A)

In FIG. 2, Garg shows that the Average Cost (A) is an output of his inventory optimization process that is derived from the set of inputs 201. In equation (14), Garg teaches that the Average Cost (A) is computed based on the inventory cost (h) and the "fixed cost of placing an order" (K) (col. 7, lines 38-39). Like holding cost, the ordering cost for a product is a well-defined parameter in the field of operations research. The attached "OR-Notes" explains that ordering costs are the costs "associated with ordering and receiving an order" (see middle of page 2). Accordingly, as with holding cost, there is no link between the current market price of the product and ordering cost. Indeed, this fact is verified by reviewing the types of costs that are included in determining holding cost (see, e.g., the middle of page 2 of the reference "OR-Notes" by J. E. Besley; attached):

- clerical/labor costs of processing orders
- inspection and return of poor quality products
- transport costs
- handling costs

Thus, neither the holding cost parameter (h) nor the ordering cost parameter (K) is linked to the current market price of the product. Therefore, the average cost parameter (A), which is computed from the holding and ordering cost parameters, also is not linked to the current market price of the product.

Therefore, contrary to the Examiner's assertion quote above, product pricing is *not* included in the average cost parameter (A).

c. The Examiner's Response To Applicants' Arguments

In the Advisory action dated September 4, 2003, the Examiner indicated that (emphasis added):

> The arguments are not persuasive as to the allowability of the claims. Applicants' arguments in reference to Garg et al. not showing product price information as an input is persuasive. However, the price input is part of the Salvo et al. combination. The rejection is still valid. The final office action was not premature. The combination of reference still meets the metes and bounds of the claims as broadly recited.

d. In Sum, Garg's System Does Not Plan The Safety Stock Level (SS) Based At Least In Part On The Market Price Of The Product

Since none of the inputs 201 into Garg's inventory optimization process includes the current market price of the product, it is not possible for the safety stock level (SS), which is computed from these inputs 201, to be planned based at least in part on the market price of the product, as acknowledged by the Examiner in the Advisory action dated September 4, 2003.

2. The Disclosure of Salvo

Salvo describes an inventory management approach in which control unit 114 determines if an inventory order is needed and then determines the lowest total inventory purchase price source (see, e.g., col. 6, lines 47-54). Salvo does not provide any details about computing a safety stock level of a product. The only teaching Salvo provides regarding inventory level computations may be summarized by the following representative disclosure: "the control unit 114 determines the amount of inventory used over time, can estimate future use, and determine if an inventory order is needed" (col. 5, lines 7-10).

In the final Office action dated June 17, 2003, the Examiner has asserted that (emphasis added):

> Salvo et al. teaches, figure 1, an inventory management system where the inventory price source module 126 searches for and compares buying options in order to optimize purchase value. The inventory price source comprises at least one of economic indicators, economic models, commodity pricing indexes, spot market pricing, Dow Jones™ information, other market information, and other inventory price sources (non-spot market pricing). The control unit 114 stores and analyzes historical trends of inventory prices to determine analyzed inventory price trends.

That is, the Examiner relies on Salvo's disclosure of an inventory price source 126 that is configured to optimize purchase value based on an analysis of price information obtained from various price sources, including spot market and non-spot market price sources. Based on this disclosure, the Examiner concluded that:

> Based on the teaching of Salvo et al., it would have been obvious to one of ordinary skill in the art, at the time the invention was made, to modify Garg et al. system to incorporate the inventory price source module of Salvo et al. as a source for many set of input values in the Garg et al. process which plans safety stock levels in order to optimize purchase value.

Contrary to the Examiner's conclusion, however, it would not have been obvious to one of ordinary skill in the art at the time of the invention to modify Garg's system by incorporating Salvo's inventory price source 126 "as a source for many set of input values in the Garg et al. process" because, as explained in detail above and acknowledged by the Examiner in the Advisory action dated September 4, 2003, Garg's process for optimizing the parameters for the (s, S) inventory policy does not have product pricing information as an input. Therefore, there is no way for Garg's optimization process to incorporate the optimized purchase price output from Salvo's inventory price source 126. Accordingly, there would not have been any motivation for one of ordinary skill in the art at the time of the invention to incorporate Salvo's inventory price source 126 into Garg's optimization process.

It is noted that the Examiner's indication that Salvo's "inventory price source module 126 searches for and compares buying options in order to optimize purchase value" does not make up for Garg's failure to teach planning a safety stock level based at least in part upon product availability from a spot market. In particular, Salvo's control unit 114 determines if an inventory order is needed based only on the amount of inventory used over time and an estimate of future use (see, e.g., col. 5, lines 7-10). That is, the step of optimizing purchase price is not used to plan inventory levels in Salvo's approach. Therefore, as in Garg's process, the fact that spot market and non-spot market pricing might be considered in determining an optimal purchase price is irrelevant to Salvo's scheme for planning safety stock levels.

3. Claim 1 Is Patentable Over Garg In View Of Salvo

For at least the reasons explained above, the Examiner's rejection of independent claim 1 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

B. Dependent claims 2-21

Each of claims 2-21 incorporates the features of independent claim 1 and, therefore, these claims are patentable for at least the same reasons explained above. Claims 2-21 also are patentable for the following additional reasons.

It is noted that in each of the Office actions dated December 20, 2002, and June 17, 2003, the Examiner has only addressed the limitations of claim 1. The Examiner consistently has failed to separately consider the limitations recited in claims 2-21 and explain his basis for rejecting these claims under 35 U.S.C. § 103(a) over Garg in view of Salvo. Therefore, the Examiner has failed to establish a proper *prima* facie case of obviousness with respect to these claims.

1. Claims 2-7, 14, and 15

Claim 2 requires the steps of (1) estimating a maximum safety stock level of the product to cover uncertainty in demand over the exposure period with the desired service level based upon product availability from a non-spot market supply, and (2) estimating an optimal safety stock level by reducing the maximum safety stock level based upon product availability from a spot market supply. Neither Garg nor Salvo, taken alone or in any permissible combination, teaches or suggests such safety stock level estimation steps.

Indeed, Garg merely discloses an inventory policy scheme in which a single safety stock level of a product (SS) needed to cover uncertainty in demand over the exposure period with the desired service level is computed. In this computation process, Garg does not distinguish among sources of supply; much less does Garg distinguish between a non-spot market supply and a spot market supply. Moreover, Garg does not even hint that the computed safety stock level (SS) could be reduced based upon product availability from a spot market supply to estimate an optimal safety stock level.

Salvo does not provide any details about computing a safety stock level of a product. As explained above, the only teaching Salvo provides regarding inventory level computations

may be summarized by the following representative disclosure: "the control unit 114 determines the amount of inventory used over time, can estimate future use, and determine if an inventory order is needed" (col. 5, lines 7-10).

Since each of Garg and Salvo fails to teach or suggest the above-mentioned features of claim 2, no permissible combination of Salvo and Garg could have taught or suggested these features to one of ordinary skill in the art at the time of the invention. Accordingly, for at least these additional reasons, the Examiner's rejection of claim 2 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

Each of claims 3-7, 14, and 15 incorporates the features of dependent claim 2 and therefore is patentable for at least the same reasons explained above.

2. <u>Claims 8, 12, and 13</u>

Each of claims 8, 12, and 13 incorporates the features of dependent claim 2 and therefore is patentable for at least the same reasons explained above. Claims 8, 12, and 13 also are patentable over Garg in view of Salvo for the following additional reasons.

Claim 8 recites that the step of "reducing the maximum safety stock level comprises estimating a total cost of covering the maximum safety stock level with a combination of product received from the spot market and product received from the non-spot market supply." Neither Garg nor Salvo teaches or suggests such a feature. Indeed, Garg does not distinguish among inventory supply sources and Salvo teaches that inventory is supplied by a single inventory product source; namely, the "lowest total inventory purchase price vendor" (col. 6, line 51). Accordingly, for at least this additional reason, the Examiner's rejection of claim 18 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

Each of claims 12 and 13 incorporates the features of claim 8 and therefore is patentable for at least the same reasons.

3. <u>Claims 9-11</u>

Claim 9 incorporates the features of claim 8 and therefore is patentable for at least the same reasons. In addition, claim 9 recites that "the total cost is estimated based in part upon an estimation of the expected amount of spot market product needed to cover uncertainty in demand over the exposure period with the desired service level for a given amount of non-spot market product." Neither Garg nor Salvo teaches or suggests anything about estimating an expected amount of product needed from a first type of product source based on a given amount of a product supplied from a different type of product source. Indeed, Garg does not

distinguish among inventory supply sources and Salvo teaches that inventory is supplied by a single inventory product source; namely, the "lowest total inventory purchase price vendor" (col. 6, line 51). Accordingly, for at least this additional reason, the Examiner's rejection of claim 18 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

Each of claims 10 and 11 incorporates the features of claim 9 and therefore is patentable for at least the same reasons.

4. Claim 16

Claims 16 incorporates the features of dependent claim 2 and therefore is patentable for at least the same reasons explained above. Claim 16 also is patentable over Garg in view of Salvo for the following additional reasons.

Claim 16 recites the step of "ordering from the spot market supply a product level needed to meet actual demand above the optimal safety stock level and within the desired service level." Neither Garg nor Salvo teaches or suggests anything about ordering a product level needed to meet actual demand *over* an optimal safety stock level, much less anything about meeting such demand with product supplied from a spot market.

5. Claim 17

Claim 17 incorporates the feature of claim 16 and therefore is patentable for at least the same reasons. Claim 17 also recites that "ordering from the spot market comprises navigating a web site providing information relating to the spot market." Neither Garg nor Salvo teaches or suggests such a feature. It is noted that the web pages shown in FIGS. 2 and 3 of Salvo, and the corresponding description at col. 11, line 18, through col. 12, line 18, relate only to graphical user interfaces for displaying actual and historical inventory levels for a receptacle at a manufacturing site. This disclosure has nothing to do with ordering product from a spot market supply.

6. Claim 18

Claim 18 recites that "planning the safety stock level comprises navigating a web site providing information relating to the use of the spot market to plan an inventory level." Neither Garg nor Salvo teaches or suggests such a feature. It is noted that the web pages shown in FIGS. 2 and 3 of Salvo, and the corresponding description at col. 11, line 18, through col. 12, line 18, relate only to graphical user interfaces for displaying actual and historical inventory levels for a receptacle at a manufacturing site. This disclosure has nothing to do with providing information relating to the use of the spot market to plan an

inventory level. Accordingly, for at least this additional reason, the Examiner's rejection of claim 18 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

7. Claim 19

Claim 19 depends from claim 18 and further recites that "planning the safety stock level comprises navigating a web site providing to an inventory planning engine accessible through the web site information relating to product demand and information relating to non-spot market lead time." Neither Garg nor Salvo teaches or suggests such a feature. It is noted that the web pages shown in FIGS. 2 and 3 of Salvo, and the corresponding description at col. 11, line 18, through col. 12, line 18, relate only to graphical user interfaces for displaying actual and historical inventory levels for a receptacle at a manufacturing site. This disclosure has nothing to do with providing information relating to product demand and information relating to non-spot market lead time to an inventory planning engine. Accordingly, for at least this additional reason, the Examiner's rejection of claim 19 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

8. Claim 20

Claim 20 recites the step of "performing enterprise resource planning based upon the planned safety stock level." Neither Garg nor Salvo teaches or suggests such a feature. Accordingly, for at least this additional reason, the Examiner's rejection of claim 20 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

9. Claim 21

Claim 21 recites that "the safety stock level is planned based at least in part on total spot market product cost as a function of amount of product supplied by one or more spot market sources." Neither Garg nor Salvo teaches or suggests such a feature. Accordingly, for at least this additional reason, the Examiner's rejection of claim 20 under 35 U.S.C. § 103(a) over Garg in view of Salvo should be withdrawn.

## IX. Conclusion

For the reasons explained above, all of the pending claims are now in condition for allowance and should be allowed.

Charge any excess fees or apply any credits to Deposit Account No. 08-2025.

Respectfully submitted,

Date: October 27, 2003

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APPENDIX

The claims that are the subject of Appeal are presented below.

1. An inventory planning method, comprising planning a safety stock level to cover uncertainty in demand over an exposure period with a desired service level based at least in part upon product availability from a spot market.

2. The method of claim 1, wherein safety stock level planning comprises:

estimating a maximum safety stock level of the product to cover uncertainty in demand over the exposure period with the desired service level based upon product availability from a non-spot market supply; and

estimating an optimal safety stock level by reducing the maximum safety stock level based upon product availability from a spot market supply.

3. The method of claim 2, wherein the maximum safety stock level estimation is based in part upon an estimation of lead time for obtaining the product from the non-spot market supply.

4. The method of claim 3, wherein the maximum safety stock level estimation is based in part upon an estimation of lead time uncertainty for obtaining the product from the non-spot market supply.

5. The method of claim 2, wherein the maximum safety stock level estimation is based in part upon an estimation of demand for the product.

6. The method of claim 5, wherein the maximum safety stock level estimation is based in part upon an estimation of demand uncertainty for the product.

7. The method of claim 2, wherein the optimal safety stock level estimation is based in part upon a cost of obtaining the product from the spot market.

8. The method of claim 2, wherein reducing the maximum safety stock level comprises estimating a total cost of covering the maximum safety stock level with a

combination of product received from the spot market and product received from the non-spot market supply.

9. The method of claim 8, wherein the total cost is estimated based in part upon an estimation of the expected amount of spot market product needed to cover uncertainty in demand over the exposure period with the desired service level for a given amount of non-spot market product.

10. The method of claim 9, wherein the total cost is estimated based in part upon a cost of obtaining the product from the spot market.

11. The method of claim 9, wherein the total cost is estimated based in part upon a cost of obtaining the product from the non-spot market supply.

12. The method of claim 8, wherein estimating the optimal safety stock level comprises minimizing the estimated total cost.

13. The method of claim 12, wherein the optimal safety stock level corresponds to a safety stock level that minimizes the estimated total cost.

14. The method of claim 2, wherein the optimal safety stock level is estimated based at least in part upon a stochastic simulation of one or more random variables.

15. The method of claim 2, further comprising ordering the optimal safety stock level from the non-spot market supply.

16. The method of claim 2, further comprising ordering from the spot market supply a product level needed to meet actual demand above the optimal safety stock level and within the desired service level.

17. The method of claim 16, wherein ordering from the spot market comprises navigating a web site providing information relating to the spot market.

18. The method of claim 1, wherein planning the safety stock level comprises navigating a web site providing information relating to use of the spot market to plan an inventory level.

19. The method of claim 18, wherein planning the safety stock level comprises providing to an inventory planning engine accessible through the web site information relating to product demand and information relating to non-spot market lead time.

20. The method of claim 1, further comprising performing enterprise resource planning based upon the planned safety stock level.

21. The method of claim 1, wherein the safety stock level is planned based at least in part on total spot market product cost as a function of amount of product supplied by one or more spot market sources.

# OR-Notes

## J E Beasley

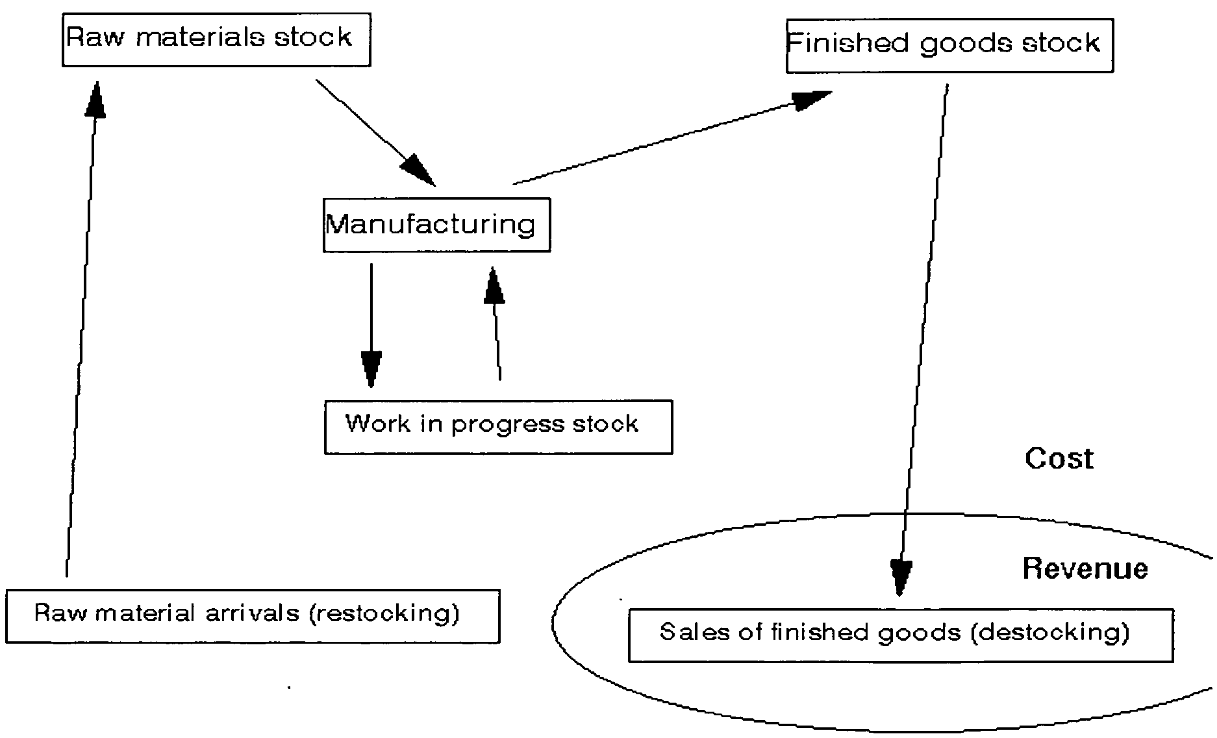
OR-Notes are a series of introductory notes on topics that fall under the broad heading of the field of operations research (OR). They were originally used by me in an introductory OR course I give at Imperial College. They are now available for use by any students and teachers interested in OR subject to the following conditions.

A full list of the topics available in OR-Notes can be found here.

---

**Inventory control**

The basic function of stock (inventory) is to insulate the production process from changes in the environment as shown below.




Note here that although we refer in this note to manufacturing, other industries also have stock e.g. the stock of money in a bank available to be distributed to customers, the stock of policemen in an area, etc).

The question then arises: *how much stock should we have?* It is this simple question that inventory control theory attempts to answer.

There are two extreme answers to this question:

*a lot*

- this ensures that we never run out
- is an easy way of managing stock
- is expensive in stock costs, cheap in management costs

*none/very little*

- this is known (effectively) as Just-in-Time (JIT)
- is a difficult way of managing stock
- is cheap in stock costs, expensive in management costs

We shall consider the problem of ordering raw material stock but the same basic theory can be applied to the problem of:

- deciding the finished goods stock; and
- deciding the size of a batch in a batch production process.

The costs that we need to consider so that we can decide the amount of stock to have can be divided into stock *holding* costs and stock *ordering* (and receiving) costs as below. **Note here that, conventionally, management costs are ignored here.**

**Holding costs - associated with keeping stock over time**

- storage costs
- rent/depreciation
- labour
- overheads (e.g. heating, lighting, security)
- money tied up (loss of interest, opportunity cost)
- obsolescence costs (if left with stock at end of product life)
- stock deterioration (lose money if product deteriorates whilst held)
- theft/insurance

**Ordering costs - associated with ordering and receiving an order**

- clerical/labour costs of processing orders
- inspection and return of poor quality products
- transport costs
- handling costs

Note here that a *stockout* occurs when we have insufficient stock to supply customers. Usually stockouts occur in the order lead time, the time between placing an order and the arrival of that order.

Given a stockout the order may be lost completely or the customer may choose to *backorder*, i.e. to be prepared to wait until we have sufficient stock to supply their order.

**Note here that whilst conceptually we can see that these cost elements are relevant it can often be difficult to arrive at an appropriate numeric figure (e.g. if the stock is stored in a building used for many other purposes, how then shall we decide an appropriate allocation of heating/lighting/security costs).**

To see how we can decide the stock level to adopt consider the very simple model below.

**Basic model**

In this basic model we have the situation where:

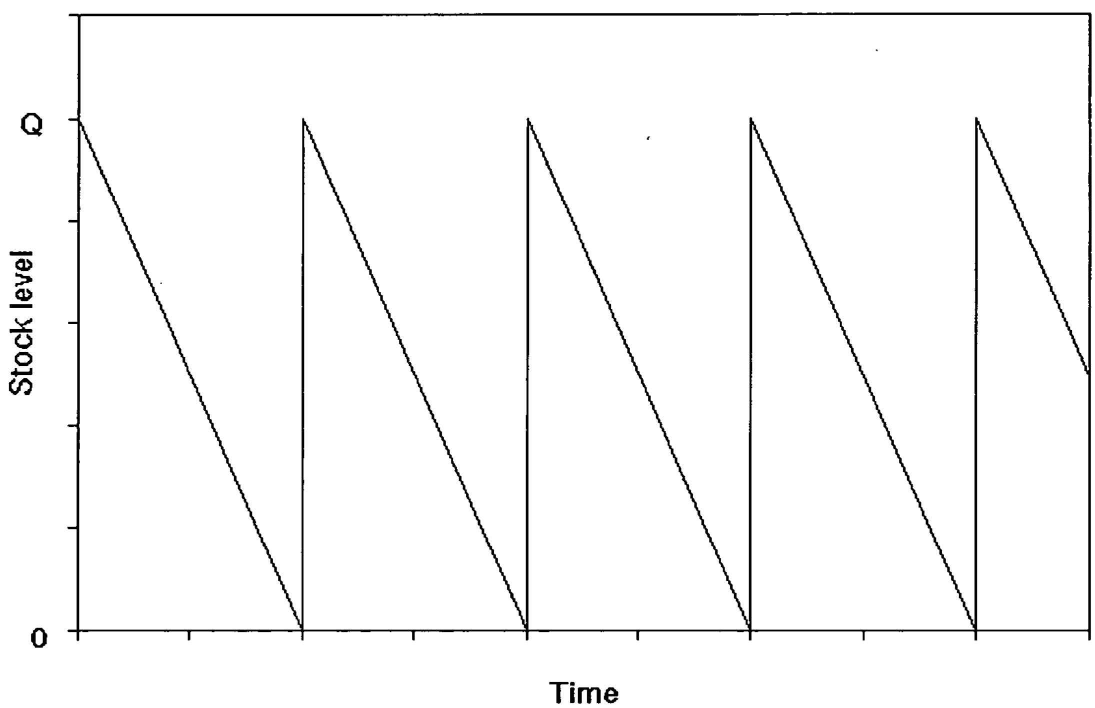
- our company orders from an outside supplier;
- that outside supplier delivers to us **precisely** the quantity we ask for; and
- we pass that stock onto our customers (either external customers, or an internal customer within the same company (e.g. if ordering raw materials for use in the production process)).

Assume:

- Stock used up at a constant rate (R units per year)
- Fixed set-up cost $c_o$ for each order - often called the order cost
- No lead time between placing an order and arrival of the order
- Variable stock holding cost $c_h$ per unit per year

Then we need to decide Q, the amount to order each time, often called the *batch* (or *lot*) size.

With these assumptions the graph of stock level over time takes the form shown below.




Note: allow stock to fall to zero as no time needed to replenish stock

Consider drawing a horizontal line at Q/2 in the above diagram. If you were to draw this line then it is

clear that the times when stock exceeds Q/2 are exactly balanced by the times when stock falls below Q/2. In other words we could equivalently regard the above diagram as representing a **constant** stock level of Q/2 over time.

Hence we have that:

- Annual holding cost = $c_h(Q/2)$

where Q/2 is the average (constant) inventory level

- Annual order cost = $c_o(R/Q)$

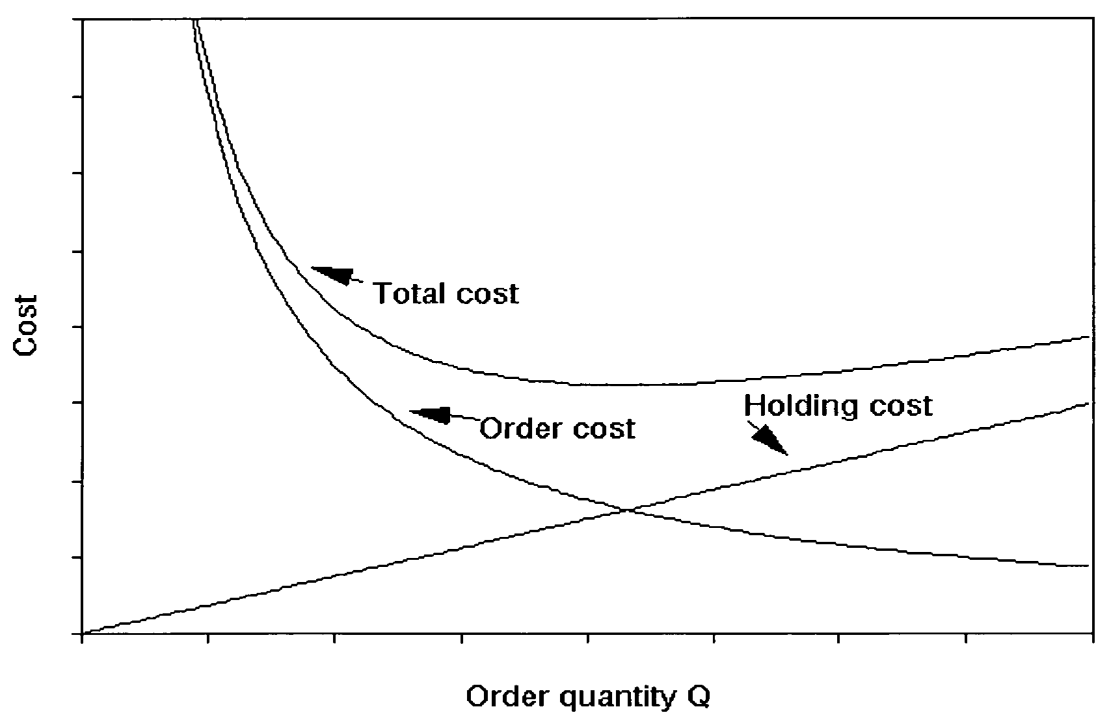
where (R/Q) is the number of orders per year (R used, Q each order)

So total annual cost = $c_h(Q/2) + c_o(R/Q)$

Total annual cost is the function that we want to *minimise* by choosing an appropriate value of Q.

Note here that, obviously, there is a purchase cost associated with the R units per year. However this is just a constant as R is fixed so we can ignore it here.

The diagram below illustrates how these two components (annual holding cost and annual order cost) change as Q, the quantity ordered, changes. As Q increases holding cost increases but order cost decreases. Hence the total annual cost curve is as shown below - somewhere on that curve lies a value of Q that corresponds to the minimum total cost.



We can calculate exactly which value of Q corresponds to the minimum total cost by differentiating total cost with respect to Q and equating to zero.

d(total cost)/dQ = $c_h/2 - c_oR/Q^2 = 0$ for minimisation

which gives $Q^2 = 2c_oR/c_h$

Hence the best value of Q (the amount to order = amount stocked) is given by

- $Q = (2Rc_o/c_h)^{0.5}$

and this is known as the *Economic Order Quantity (EOQ)*

**Comments**

This formula for the EOQ is believed to have been first derived in the early 1900's and so EOQ dates from the beginnings of mass production/assembly line production.

To get the total annual cost associated with the EOQ we have from before that total annual cost = $c_h(Q/2) + c_o(R/Q)$ so putting $Q = (2Rc_o/c_h)^{0.5}$ into this we get that the total annual cost is given by

$$c_h((2Rc_o/c_h)^{0.5}/2) + c_o(R/(2Rc_o/c_h)^{0.5}) = (Rc_oc_h/2)^{0.5} + (Rc_oc_h/2)^{0.5} = (2Rc_oc_h)^{0.5}$$

Hence total annual cost is $(2Rc_oc_h)^{0.5}$ which means that when ordering the optimal (EOQ) quantity we have that total cost is proportional to the square root of any of the factors (R, $c_o$ and $c_h$) involved. For example, if we were to reduce $c_o$ by a factor of 4 we would reduce total cost by a factor of 2 (note the EOQ would change as well). This, in fact, is the basis of Just-in-Time (JIT), to reduce (continuously) $c_o$ and $c_h$ so as to drive down total cost.

To return to the issue of management costs being ignored for a moment the basic justification for this is that if we consider the total cost curve shown above, then - assuming we are not operating a policy with a very low Q (JIT) or a very high Q - we could argue that the management costs are effectively fixed for a fairly wide range of Q values. If this is so then such costs would not influence the decision as to what order quantity Q to adopt. Moreover if we wanted to adopt a more quantitative approach we would need some function that captures the relationship between the management costs we incur and our order quantity Q - estimating this function would certainly be a non-trivial task.

---

**Example**

A retailer expects to sell about 200 units of a product per year. The storage space taken up in his premises by one unit of this product is costed at £20 per year. If the cost associated with ordering is £35 per order what is the economic order quantity given that interest rates are expected to remain close to 10% per year and the total cost of one unit is £100.

We use the EOQ formula,

$EOQ = (2Rc_o/c_h)^{0.5}$

Here R=200, $c_o$=35 and the holding cost $c_h$ is given by

$c_h$ = £20 (direct storage cost per unit per year) + £100 x 0.10 (this term the money interest lost if one unit sits in stock for one year)

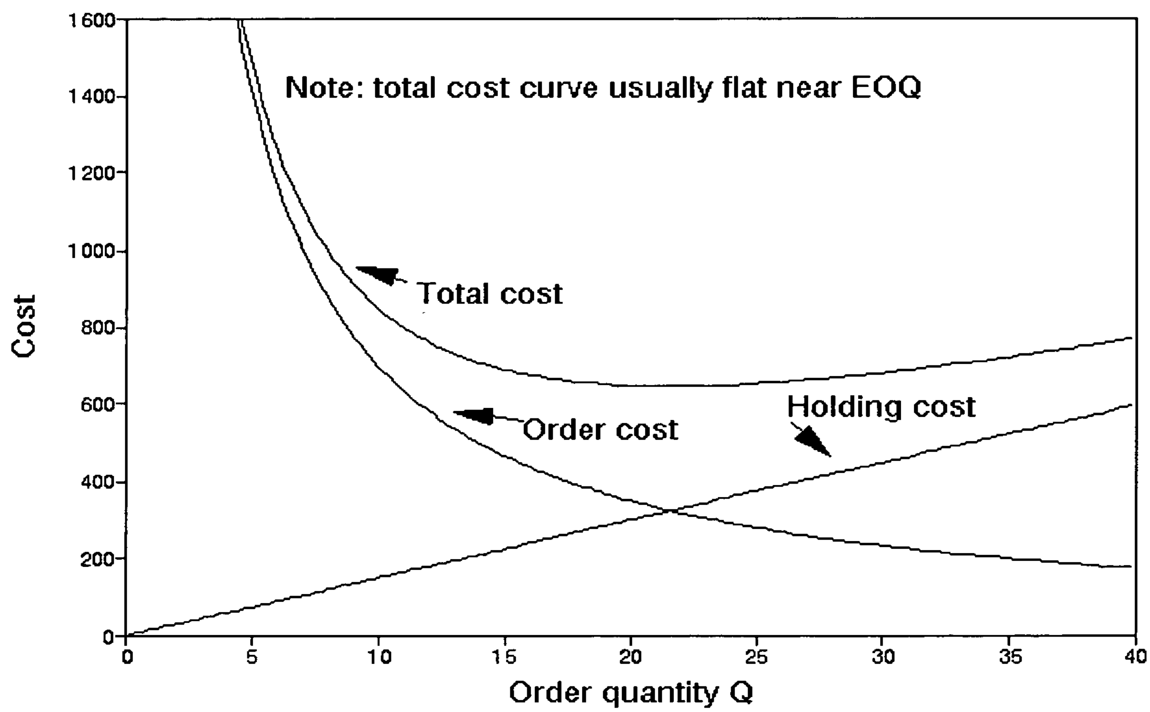
i.e. $c_h$ = £30 per unit per year

Hence $EOQ = (2Rc_o/c_h)^{0.5} = (2 \times 200 \times 35/30)^{0.5} = 21.602$

But as we must order a whole number of units we have that:

EOQ = 22

We can illustrate this calculation by reference to the diagram below which shows order cost, holding cost and total cost for this example.




With this EOQ we can calculate our total annual cost from the equation

Total annual cost = $c_h(Q/2) + c_o(R/Q)$

Hence for this example we have that

Total annual cost = (30 x 22/2) + (35 x 200/22) = 330 + 318.2 = £648.2

**Note:** If we had used the exact Q value given by the EOQ formula (i.e. Q=21.602) we would have had that the two terms relating to annual holding cost and annual order cost would have been exactly equal to each other

i.e. holding cost = order cost at EOQ point (or, referring to the diagram above, the EOQ quantity is at the point associated with the Holding Cost curve and the Order Cost curve intersecting).

i.e. $(c_hQ/2) = (c_oR/Q)$ so that $Q = (2Rc_o/c_h)^{0.5}$

**In other words, as in fact might seem natural from the shape of the Holding Cost and Order Cost curves, the optimal order quantity coincides with the order quantity that exactly balances Holding Cost and Ordering Cost.**

Note however that this result only applies to certain simple situations. It is not true (in general) that the best order quantity corresponds to the quantity where holding cost and ordering cost are in balance.

---

**Package solution**

We can also solve this problem using the package, the input and output being shown below. Note here that the package can deal with more complicated factors than we have considered in the simple example given above.

| 09-19-2000 | Input Data | Value | Economic Order Analysis | Value |
|---|---|---|---|---|
| 1 | Demand per year | 200 | Order quantity | 21.6025 |
| 2 | Order (setup) cost | £35.00 | Maximum inventory | 21.6025 |
| 3 | Unit holding cost per year | £30.00 | Maximum backorder | 0 |
| 4 | Unit shortage cost | | Order interval in year | 0.1080 |
| 5 | per year | M | Reorder point | 0 |
| 6 | Unit shortage cost | | | |
| 7 | independent of time | 0 | Total setup or ordering cost | £324.04 |
| 8 | Replenishment/production | | Total holding cost | £324.04 |
| 9 | rate per year | M | Total shortage cost | 0 |
| 10 | Lead time in year | 0 | Subtotal of above | £648.07 |
| 11 | Unit acquisition cost | £100.00 | | |
| 12 | | | Total material cost | £20,000.00 |
| 13 | | | | |
| 14 | | | Grand total cost | £20,648.07 |

Note the appearance here of the figure of 20,000 relating to material cost. This is calculated from using 200 units a year at a unit cost of £100 each. Strictly, this cost term should have been added to the total annual cost equation $(c_h(Q/2) + c_o(R/Q))$ we gave above. We neglected it above as it was a constant term for this example and hence did not affect the calculation of the optimal value of Q. However, we will need to remember to include this term below when we come to consider quantity discounts.

---

**Example**

Suppose, for administrative convenience, we ordered 20 and not 22 at each order - what would be our cost penalty for deviating from the EOQ value?

With a Q of 20 we look at the total annual cost

$= (c_h Q/2) + (c_o R/Q)$

$= (30 \times 20)/2 + (35 \times 200/20) = 300 + 350 = £650$

Hence the cost penalty for deviating from the EOQ value is £650 - £648.2 = £1.8

Note that this is, relatively, a very small penalty for deviating from the EOQ value. This is usually the case in inventory problems i.e. the total annual cost curve is flat near the EOQ so there is only a small cost penalty associated with slight deviations from the EOQ value (see the diagram above).

This is an important point. Essentially we should view the EOQ as a **ballpark** figure. That is it gives us a rough idea as to how many we should be ordering each time. After all our cost figures (such as cost of an order) are likely to be inaccurate. Also it is highly unlikely that we will use items at a constant rate (as the EOQ formula assumes). However, that said, the EOQ model provides a systematic and quantitative way of getting an idea as to how much we should order each time. If we deviate far from this ballpark figure then we will most likely be paying a large cost penalty.

The above cost calculation can also be done using the package - see below.

| Economic Order Analysis | Value | Known Order Analysis | Value |
|---|---|---|---|
| Order quantity | 21.6025 | Order quantity | 20 |
| Maximum inventory | 21.6025 | Maximum inventory | 20 |
| Maximum backorder | 0 | Maximum backorder | 0 |
| Order interval in year | 0.1080 | Order interval in year | 0.1 |
| Reorder point | 0 | Reorder point | 0 |
| | | | |
| Total setup or ordering cost | £324.04 | Total setup or ordering cost | £350.00 |
| Total holding cost | £324.04 | Total holding cost | £300.00 |
| Total shortage cost | 0 | Total shortage cost | 0 |
| Subtotal of above | £648.07 | Subtotal of above | £650.00 |
| | | | |
| Total material cost | £20,000.00 | Total material cost | £20,000.00 |
| | | | |
| Grand total cost | £20,648.07 | Grand total cost | £20,650.00 |

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**Extensions**

In order to illustrate extensions to the basic EOQ calculation we will consider the following example:

A company uses 12,000 components a year at a cost of 5 pence each. Order costs have been estimated to be £5 per order and inventory holding cost is estimated at 20% of the cost of a component per year.

Note here that this is the sort of cheap item that is a typical non-JIT item.

- **What is the EOQ?**

Here R=12000, $c_o$=5 and as the inventory holding cost is 20% per year the annual holding cost per unit $c_h$ = cost per unit x 20% = £0.05 x 0.2 per unit per year = 0.01.

Hence EOQ = $(2Rc_o/c_h)^{0.5}$ = $(2 \times 12000 \times 5/0.01)^{0.5}$ = 3464

The package output for this problem is shown below.

| 09-19-2000 | Input Data | Value | Economic Order Analysis | Value |
|---|---|---|---|---|
| 1 | Demand per year | 12000 | Order quantity | 3464.102 |
| 2 | Order (setup) cost | £5.00 | Maximum inventory | 3464.102 |
| 3 | Unit holding cost per year | £0.01 | Maximum backorder | 0 |
| 4 | Unit shortage cost | | Order interval in year | 0.2887 |
| 5 | per year | M | Reorder point | 0 |
| 6 | Unit shortage cost | | | |
| 7 | independent of time | 0 | Total setup or ordering cost | £17.32 |
| 8 | Replenishment/production | | Total holding cost | £17.32 |
| 9 | rate per year | M | Total shortage cost | 0 |
| 10 | Lead time in year | 0 | Subtotal of above | £34.64 |
| 11 | Unit acquisition cost | £0.05 | | |
| 12 | | | Total material cost | £600.00 |
| 13 | | | | |
| 14 | | | Grand total cost | £634.64 |

---

- **If orders must be made for 1,2,3,4,6 or 12 monthly batches what order size would you recommend and when would you order?**

Here we do not have an unrestricted choice of order quantity (as the EOQ formula assumes) but a restricted choice as explained below.

This is an *important* point - the EOQ calculation gives us a *quantity* to order, but often people are better at ordering on a *time* basis e.g. once every month.

***In other words we need to move from a quantity basis to a time basis.***

For example the EOQ quantity of 3464 has an order interval of (3464/12000) = 0.289 years, i.e. we order once every 52(0.289) = 15 weeks. Would you prefer to order once every 15 weeks or every 4 months? Recall here that we saw before that small deviations from the EOQ quantity lead to only small cost changes.

Hence if orders must be made for 1,2,3,4,6 or 12 monthly batches the best order size to use can be determined as follows.

Obviously when we order a batch we need only order sufficient to cover the number of components we are going to use until the next batch is ordered - if we order less than this we will run out of components and if we order more than this we will incur inventory holding costs unnecessarily. Hence for each possible batch size we automatically know the order quantity (e.g. for the 1-monthly batch the order quantity is the number of components used per month = R/12 = 12000/12 = 1000).

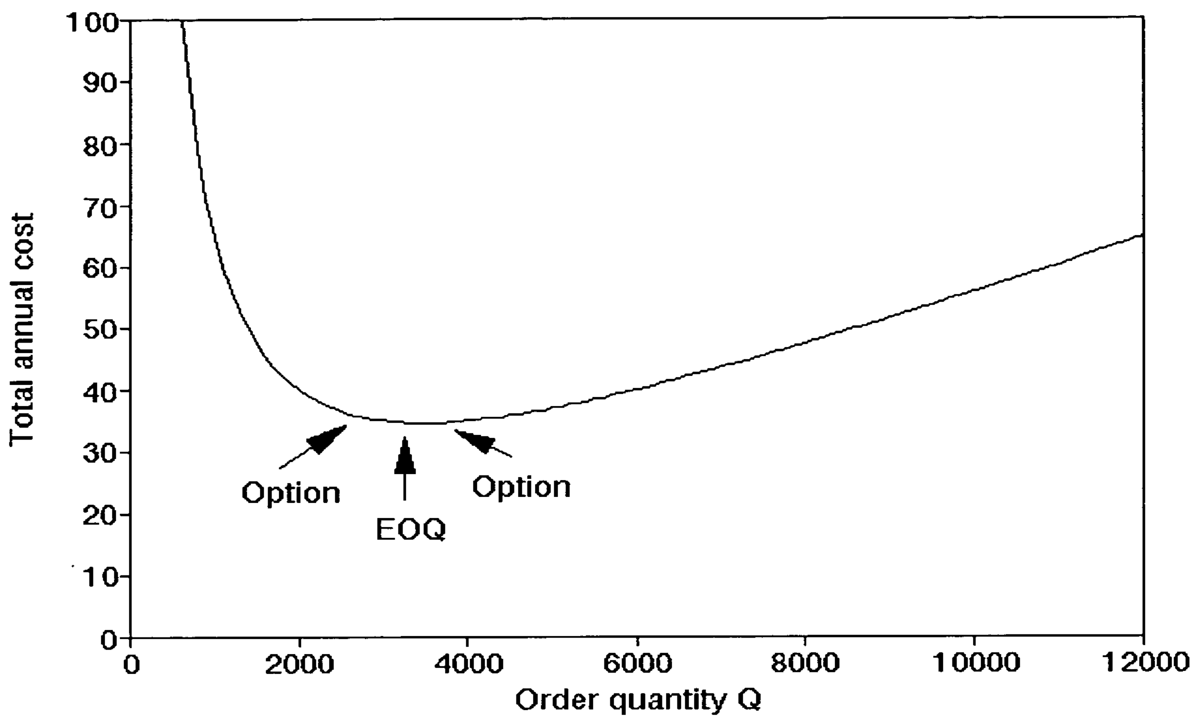
As we know the order quantity we can work out the total annual cost of each of the different options and choose the cheapest option.

The total annual cost (with an order quantity of Q) is given by $(c_hQ/2) + (c_oR/Q)$ and we have the table below:

| Batch size option | Order quantity Q | Total annual cost |
|---|---|---|
| Monthly | 1000 | 65 |
| 2-monthly | 2000 | 40 |
| 3-monthly | 3000 | 35 |
| 4-monthly | 4000 | 35 |
| 6-monthly | 6000 | 40 |
| 12-monthly | 12000 | 65 |

The least cost option therefore is to choose either the 3-monthly or the 4-monthly batch.

In fact we need not have examined all the options. As we knew that the EOQ was 3464 (associated with the minimum total annual cost) we have that the least cost option must be one of the two options that have order quantities *nearest* to 3464 (one order quantity above 3464, the other below 3464) i.e. *either* the 3-monthly (Q=3000) *or* the 4-monthly (Q=4000) options. This can be seen from the shape of the total annual cost curve shown below. The total annual cost for these two options could then be calculated to find which was the cheapest option.

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- **If the supplier offers the following quantity discount structure what effect will this have on the order quantity?**

| Order quantity | Cost (per unit) |
|---|---|
| 0-4,999 | £0.05 |
| 5,000-9,999 | £0.05 less 5% |
| 10,000-19,999 | £0.05 less 10% |
| 20,000 and above | £0.05 less 15% |

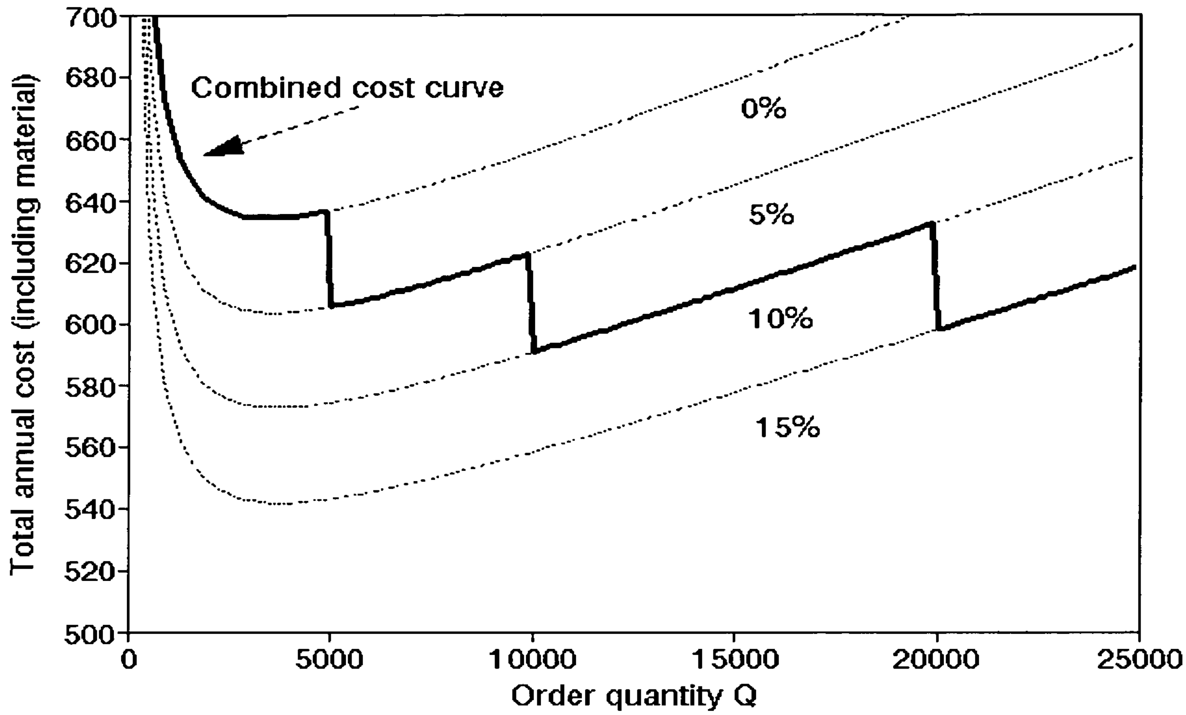
For example, were we to order 6000 units we would only pay 0.95(0.05) for each and every one of the 6000 units, i.e. the discount would be given on the entire order.

Here, as mentioned above, we need to remember to add to the total annual cost equation ($c_h(Q/2) + c_o(R/Q)$) a term relating to R multiplied by the unit cost, as the cost of a unit is now no longer fixed but variable (unit cost = a function f(Q) of the order quantity Q). Hence our total annual cost equation is

$$c_h(Q/2) + c_o(R/Q) + R[f(Q)]$$

It is instructive to consider what changes in this equation as we change the order quantity Q. Obviously R and $c_o$ remain unchanged, equally obviously Q and f(Q) change. So what of $c_h$? Well it can remain constant or it can change. You need to look back to how you calculated $c_h$. If it included money tied up then, as the unit cost f(Q) alters with Q, so too does the money tied up.

The effect of these quantity discounts (breaks in the cost structure) is to create a discontinuous total annual cost curve as shown below with the total annual cost curve for the combined discount structure being composed of parts of the total annual cost curves for each of the discount costs.




**The order quantity which provides the lowest overall cost will be the lowest point on the Combined Cost Curve shown in the diagram above.** We can precisely calculate this point as it corresponds to:

- **either** an EOQ for one of the discount curves considered separately (note that in some cases the EOQ for a particular discount curve may not lie within the range covered by that discount and hence will be infeasible);
- **or** one of the breakpoints between the individual discount curves on the total annual cost curve for the combined discount structure.

We merely have to work out the total annual cost for each of these types of points and choose the cheapest.

First the EOQ's:

| Discount | Cost | $c_h$ | EOQ | Inventory cost | Material cost | Total cost |
|---|---|---|---|---|---|---|
| 0 | 0.05 | 0.01 | 3464 | 34.64 | 600 | 634.64 |
| 5% | 0.0475 | 0.0095 | 3554 | Infeasible | | |
| 10% | 0.045 | 0.009 | 3651 | Infeasible | | |
| 15% | 0.0425 | 0.0085 | 3757 | Infeasible | | |

Note here that we now include *material (purchase)* cost in total annual cost.

The effect of the discount is to reduce the cost, and hence $c_h$ the inventory holding cost per unit per year - all other terms in the EOQ formula (R and $c_o$) remain the same. Of the EOQ's only one, the first, lies within the range covered by the discount rate.

For the breakpoints we have:

| Order quantity | Cost | $c_h$ | Inventory cost | Material cost | Total cost |
|---|---|---|---|---|---|
| 5,000 | 0.0475 | 0.0095 | 35.75 | 570 | 605.75 |
| 10,000 | 0.045 | 0.009 | 51 | 540 | 591 |
| 20,000 | 0.0425 | 0.0085 | 88 | 510 | 598 |

From these figures we can see that the economic order quantity associated with minimum total annual cost is 10,000 with a total annual cost of 591.

Note too here that this situation illustrates the point we made before when we considered the simple EOQ model, namely that it is not true (in general) that the best order quantity corresponds to the quantity where holding cost and ordering cost are in balance. This is because the holding cost associated with Q=10,000 is $c_h(Q/2) = 0.009(10000/2) = 45$, whilst the ordering cost is $c_o(R/Q) = 5(12000/10000) = 6$.

This problem can also be solved using the package - the input being shown below. Note that in the package one needs to use Edit Discount Breaks to enter the discount structure.

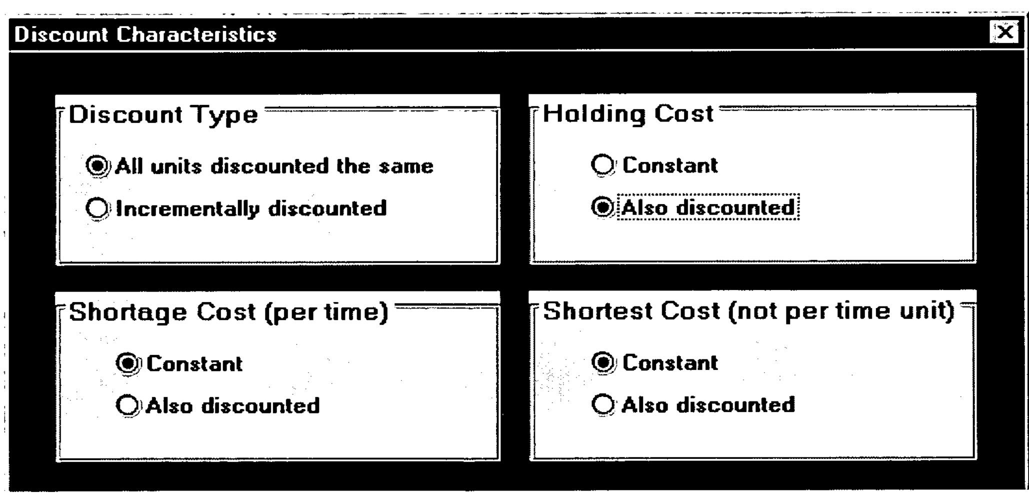
| DATA ITEM | ENTRY |
|---|---|
| Demand per year | 12000 |
| Order or setup cost per order | 5 |
| Unit holding cost per year | 0.01 |
| Unit shortage cost per year | M |
| Unit shortage cost independent of time | |
| Replenishment or production rate per year | M |
| Lead time for a new order in year | |
| Unit acquisition cost without discount | 0.05 |
| Number of discount breaks (quantities) | 3 |
| Order quantity if you known | |

Discount Breaks

5000

| Number | Discount Break | Discount % |
|---|---|---|
| 1 | 5000 | 5 |
| 2 | 10000 | 10 |
| 3 | 20000 | 15 |

One also needs to use Edit Discount Characteristics. Below we have specified that holding cost is also discounted as the cost of an item changes.




The output from the package is shown below. Notice in that output how the package is considering the same choices (EOQ's and breakpoints) as in our manual calculation above.

| 09-19-2000 | Break Qty. | Discount % | EOQ | EOQ Cost | Feasibility | Order Qty. | Total Cost |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 3464.102 | £634.64 | Yes | 3464.102 | £634.64 |
| 1 | 5000 | 5 | 3554.093 | £603.76 | No | 5000 | £605.75 |
| 2 | 10000 | 10 | 3651.484 | £572.86 | No | 10000 | £591.00 |
| 3 | 20000 | 15 | 3757.346 | £541.94 | No | 20000 | £598.00 |
| | Recommended | Order Qty. = | 10000 | Discount = | 10% | Total Cost = | £591.00 |

Note here the use of discount analysis is not restricted to buyers, it can also be used by a supplier to investigate the likely effects upon the orders he receives of changes in the discount structure. For example if the supplier lowers the order size at which a particular discount is received then how might this effect the orders he receives - will they become bigger/smaller, less frequent/more frequent?

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**Newsvendor problem**

Consider a newsvendor who stands on the street and sells an evening paper, the Evening News. How many copies should he stock?

He sells the paper to his customers for 35 (pence) a copy. He pays his supplier 20 (pence) a copy, but any unsold copies can be returned to the supplier and he gets 10 (pence) back. This is known as a *salvage value*. Assume that his demand for copies on any day is a Normal distribution of mean 100 and standard deviation 7.

Before we can compute the amount he should order we need to work out his shortage cost per unit - how much does he lose if a customer wants a copy and he does not have a copy available?

As a first analysis he loses his profit (= revenue - cost = 35 - 20 =15) so we can *estimate* his shortage cost (opportunity cost) as 15 (this ignores any loss of goodwill and any loss of future custom that might result from a shortage).

Giving this information to the package we get:

| 11-03-2000 | Input Data or Result | Value |
|---|---|---|
| 1 | Demand distribution (in day) | Normal |
| 2 | Demand mean | 100 |
| 3 | Demand standard deviation | 7 |
| 4 | Order or setup cost | 0 |
| 5 | Unit cost | £0.20 |
| 6 | Unit selling price | £0.35 |
| 7 | Unit shortage (opportunity) cost | £0.15 |
| 8 | Unit salvage value | £0.10 |
| 9 | Initial inventory | 0 |
| 10 | | |
| 11 | Optimal order quantity | 104.7228 |
| 12 | Optimal inventory level | 104.7228 |
| 13 | Optimal service level | 75% |
| 14 | Optimal expected profit | £14.11 |

This tells us he should stock 104.7 (say 105) copies of the paper. This service level of 75% means that, on average, he will be able to completely supply his customer on 75 days in every 100, i.e. 3 days out of 4. The remainder of the time (1 day out of 4) he will experience shortages, some customers will not be able to buy a copy from him as he will have run out.

More sophisticated variants of this simple model can be used, for example, to decide how many copies of a magazine to have on a shelf in a newsagent (such as W.H.Smith).

Note that there is an important conceptual difference between this newsvendor problem and the EOQ/discount problems considered above. In those EOQ/discount problems we had a decision problem (how much to order) even though the situation was one of certainty - we knew precisely the rate at which we used items. In the newsvendor problem if we knew for certain how many customers will want a paper each day then the decision problem becomes trivial (order exactly that many). In other words:

- **for the EOQ problem we had a decision problem even though there was no uncertainty**
- **for the newsvendor problem it was only the uncertainty that created the decision problem**

---

### Comment

There are many extensions to the simple EOQ models we have considered - for example:

- reorder lead time - allow a lead time between placing an order and receiving it - this introduces the problem of when to reorder (typically at some stock level called the *reorder level*). You will see from the package input that the package allows for a reorder lead time.
- stockouts - we can allow *stockouts* (often called *shortages*) i.e. no stock currently available to meet orders. You will see from the package input that the package allows for shortage costs.
- often an order is not received all at once, for example if the order comes from another part of the same factory then items may be received as they are produced. You will see from the package input that the package allows for a *replenishment* or *production* rate.
- buffer (safety) stock - some stock kept back to be used only when necessary to prevent stockouts.